(8 Pages) **Reg. No. : ....**

Code No. : 6845 **Sub. Code : PMAM 32**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Third Semester

Core — Mathematics

TOPOLOGY — I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Which one of the following is a topology on $X = \{a,b,c\}$

   (a) $\{\phi, X, \{a\}, \{b\}\}$

   (b) $\{\phi, X, \{a,b\}, \{b,c\}\}$

   (c) $\{\phi, \{a\}, \{b\}, \{a,b\}\}$

   (d) $\{\phi, X, \{a,c\}\}$

2. Which one of the following is not true

(a) If $A = [0, 1]$, then $A' = [0, 1]$

(b) If $B = \left\{ \frac{1}{n} / n \in Z_+ \right\}$ then $B' = \{0\}$

(c) If $C = Q$ then $C' = R$

(d) If $D = Z_+$ then $D' = Z_+$

3. π_3 (7, 8, 9, 2, 3, 5) is

(a) 9 (b) 8

(c) 2 (d) 3

4. If $f : R \to R$ is defined by $f(x) = 3x + 1$ then f^{-1} is given by

(a) $f^{-1}(y) = \frac{y+1}{3}$ (b) $f^{-1}(y) = \frac{y-1}{3}$

(c) $f^{-1}(y) = 3y - 1$ (d) $f^{-1}(y) = \frac{y}{3} - 1$

5. If d is the discrete metric on X, which one of the following is not true (Here $a \in X$ and $|X| \geq 2$)

(a) $B(a, 1/2) = \{a\}$ (b) $B(a, 1) = X$

(c) $B(a, 2) = X$ (d) $B(a, 0.8) = \{a\}$

6. Which one of the following is of true

(a) R^n in the product topology is metrizable

(b) R^w in the product topology is metrizable

(c) The uniform topology on R^J is metrizable

(d) R^w is the box topology in metrizable

7. Which one of the following set is connected in R

(a) $(2, 5) \cup (7, 9)$

(b) $(2, 6) \cup (5, 10)$

(c) $\{0\} \cup (1, 2)$

(d) Q (the set of all ration number)

8. Which one of the following is not true?

(a) every closed subspace of a compact space is compact

(b) every compact subspace of a Hausdorff space is closed

(c) any space containing only finitely many points is compact

(d) R is compact

9. A space X is said to be limit point compact it

(a) every finite subset of X has a limit point

(b) every infinite subset of X is bounded

(c) every infinite subset of X has a limit point

(d) every subset of X has a limit point

10. Which one of the following is not locally compact

(a) the real line $\mathbb{R}$

(b) the subspace Q of rational numbers

(c) the space $\mathbb{R}^n$

(d) every simply ordered set having the l.u.b. property

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Let B and B' be bases for the topologies τ and τ' respectively on X. Prove that τ' is fines than τ if and only if for each $x \in X$ and each basis element $B \in B$ containing x, there is a basis element $B' \in B'$ such that $x \in B' \subseteq B$.

Or

(b) Define a Hausdorff space. Prove that every finite point set in a Hausdorff space is closed.

12. (a) If B is a basis for the topology of X and e is a basis for the topology of Y, prove that the collection $D = \{B \times C / B \in B \text{ and } C \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$.

Or

(b) State and prove the pasting lamma.

13. (a) Let X be a metric space with metric d. Define $\bar{d} : X \times X \to R$ by the equation $\bar{d}(x, y) = \min\{d(x, y), 1\}$. Prove that $\bar{d}$ is a metric that induces the same topology as d.

Or

(b) Let $f : X \to Y$. If the function f is continuous, prove that for every convergent sequence $x_n \to x$ in X, the sequence $f(x_n)$ converges to $f(x)$. Also show that the converse holds if X is metrizable.

14. (a) If the sets C and D form a separation of X, and if Y is a connected subspace of X, prove that Y lies entirely within either C on D.

Or

(b) Prove that the image of a compact space under a continuous map is compact.

15. (a) Define a limit point compact space. Give an example of a limit point compact space which is not compact with justification.

Or

(b) Let X be a locally compact Hausdorff; let A be a subspace of X. If A is closed in X or open in X, prove that A is locally compact.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Define a topology. Find two non comparable topologies on $X = \{a, b, c\}$. If $\{\tau_\alpha\}$ is a family of topologies on X, show that $\bigcap \tau_\alpha$ is a topology on X. Is $\bigcup \tau_\alpha$ a topology on X? Justify.

Or

(b) (i) Let Y be a subspace of X. Prove that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.

(ii) If X is a Hawsdorff space, prove that a sequence of points of X converges to at most one point of X.

17. (a) State and prove any three rules for constructing continuous functions.

Or

(b) Define the box and product topologies and compare them.

18. (a) Define a suitable metric D on R^w and show that it induces the product topology on R^w.

Or

(b) State and prove the uniform limit theorem.

19. (a) (i) Prove that the union of a collection of connected subspaces of X that have a point in common is connected.

(ii) Show that if X is an infinite set, it is connected in the finite complement topology.

Or

(b) Prove that the product of finitely many compact spaces is compact.

20. (a) Define a sequentially compact space and show that every limit point compact space is sequentially compact if X is metrizable.

Or

(b) If X is a locally compact Hausdorff space that is not itself compact, prove that X has a one-point compactification.